MODULE 4 : HYPOTHESIS

- Hypothesis is usually considered as the principal instrument in research.
- Its main function is to suggest new experiments and observations.
- Many experiments are carried out with the deliberate object of testing hypotheses.
- Decision-makers often face situations wherein they are interested in testing hypotheses on the basis of available information and then take decisions on the basis of such testing.
- In social science, where direct knowledge of population parameter(s) is rare, hypothesis testing is the often used strategy for deciding whether a sample data offer such support for a hypothesis that generalization can be made.

- Thus hypothesis testing enables us to make probability statements about population parameter(s).
- The hypothesis may not be proved absolutely, but in practice it is accepted if it has withstood a critical testing.
- Before we explain how hypotheses are tested through different tests meant for the purpose, it will be appropriate to explain clearly the meaning of a hypothesis and the related concepts for better understanding of the hypothesis testing techniques

WHAT IS A HYPOTHESIS?

- Ordinarily, when one talks about hypothesis, one simply means a mere assumption or some supposition to be proved or disproved.
- But for a researcher hypothesis is a formal question that he intends to resolve.
- Thus a hypothesis may be defined as a proposition or a set of proposition set forth as an explanation for the occurrence of some specified group of phenomena either asserted merely as a provisional conjecture to guide some investigation or accepted as highly probable in the light of established facts.
- Quite often a research hypothesis is a predictive statement, capable of being tested by scientific methods, that relates an independent variable to some dependent variable

WHAT IS A HYPOTHESIS?

- For example, consider statements like the following ones:
- “Students who receive counselling will show a greater increase in creativity than students not receiving counselling” Or “the automobile *A* is performing as well as automobile *B*.”
- These are hypotheses capable of being objectively verified and tested.
- Thus, we may conclude that a hypothesis states what we are looking for and it is a proposition which can be put to a test to determine its validity.

Characteristics of hypothesis

Hypothesis must possess the following characteristics:

(i) Hypothesis should be clear and precise. If the hypothesis is not clear and precise, the inferences drawn on its basis cannot be taken as reliable.

(ii) Hypothesis should be capable of being tested. In a swamp of untestable hypotheses, many a time the research programmes have bogged down. Some prior study may be done by researcher in order to make hypothesis a testable one. A hypothesis "is testable if other deductions can be made from it which, in turn, can be confirmed or disproved by observation."

(iii) Hypothesis should state relationship between variables, if it happens to be a relational hypothesis.

(iv) Hypothesis should be limited in scope and must be specific. A researcher must remember that narrower hypotheses are generally more testable and he should develop such hypotheses.

(v)Hypothesis should be stated as far as possible in most simple terms so that the same is easily understandable by all concerned. But one must remember that simplicity of hypothesis has nothing to do with its significance.

(vi) Hypothesis should be consistent with most known facts i.e., it must be consistent with a substantial body of established facts. In other words, it should be one which judges accept as being the most likely.

(vii) Hypothesis should be amenable to testing within a reasonable time. One should not use even an excellent hypothesis, if the same cannot be tested in reasonable time for one cannot spend a life-time collecting data to test it.

(viii) Hypothesis must explain the facts that gave rise to the need for explanation. This means that by using the hypothesis plus other known and accepted generalizations, one should be able to deduce the original problem condition.

Thus hypothesis must actually explain what it claims to explain; it should have empirical reference.

BASIC CONCEPTS CONCERNING TESTING OF HYPOTHESES

Basic concepts in the context of testing of hypotheses need to be explained.

(a) Null hypothesis and alternative hypothesis:

- In the context of statistical analysis, we often talk about null hypothesis and alternative hypothesis.
- If we are to compare method *A* with method *B* about its superiority and if we proceed on the assumption that both methods are equally good, then this assumption is termed as the null hypothesis.
- As against this, we may think that the method *A* is superior or the method *B* is inferior, we are then stating what is termed as alternative hypothesis.
- The null hypothesis is generally symbolized as H_0 and the alternative hypothesis as *Ha*.

- Suppose we want to test the hypothesis that the population mean(μ) is equal to the hypothesized mean (μ_{H0}) = 100
- Then we would say that the null hypothesis is that the population mean is equal to the hypothesized mean 100 and symbolically we can express as:
- $H0 : \mu = \mu_{H0} = 100$
- If our sample results do not support this null hypothesis, we should conclude that something else is true. What we conclude rejecting the null hypothesis is known as alternative hypothesis.
- In other words, the set of alternatives to the null hypothesis is referred to as the alternative hypothesis.
- If we accept *H*0, then we are rejecting *Ha* and if we reject *H*0, then we are accepting *Ha*.

- For H0 : $\mu = \mu_{H0} = 100$, we may consider three possible alternative hypotheses as follows* :

Table 9.1

Alternative hypothesis	*To be read as follows*
$H_a : \mu \neq \mu_{H_0}$	(The alternative hypothesis is that the population mean is not equal to 100 i.e., it may be more or less than 100)
$H_a : \mu > \mu_{H_0}$	(The alternative hypothesis is that the population mean is greater than 100)
$H_a : \mu < \mu_{H_0}$	(The alternative hypothesis is that the population mean is less than 100)

**If a hypothesis is of the type $m = m_{H0}$, then we call such a hypothesis as simple (or specific) hypothesis but if it is of the type $m \neq \mu_{H0}$ $m > m_{H0}$ or $m < \mu_{H0}$, then we call it a composite (or nonspecific) hypothesis.*

The null hypothesis and the alternative hypothesis are chosen before the sample is drawn (the researcher must avoid the error of deriving hypotheses from the data that he collects and then testing the hypotheses from the same data).

In the choice of null hypothesis, the following considerations are usually kept in view:

- (a) Alternative hypothesis is usually the one which one wishes to prove and the null hypothesis is the one which one wishes to disprove. Thus, a null hypothesis represents the hypothesis we are trying to reject, and alternative hypothesis represents all other possibilities.
- (b) If the rejection of a certain hypothesis when it is actually true involves great risk, it is taken as null hypothesis because then the probability of rejecting it when it is true is a (the level of significance) which is chosen very small.
- (c) Null hypothesis should always be specific hypothesis i.e., it should not state about or approximately a certain value.

- Generally, in hypothesis testing we proceed on the basis of null hypothesis, keeping the alternative hypothesis in view.
- Why so? The answer is that on the assumption that null hypothesis is true, one can assign the probabilities to different possible sample results, but this cannot be done if we proceed with the alternative hypothesis.
- Hence the use of null hypothesis (at times also known as statistical hypothesis) is quite frequent.

(b) The level of significance:

- This is a very important concept in the context of hypothesis testing.
- It is always some percentage (usually 5%) which should be chosen wit great care, thought and reason.
- In case we take the significance level at 5 per cent, then this implies that H_0 will be rejected when the sampling result (i.e., observed evidence) has a less than 0.05 probability of occurring if H_0 is true.
- In other words, the 5 per cent level of significance means that researcher is willing to take as much as a 5 per cent risk of rejecting the null hypothesis when it (H_0) happens to be true.
- Thus the significance level is the maximum value of the probability of rejecting H_0 when it is true and is usually determined in advance before testing the hypothesis.

(c) Decision rule or test of hypothesis:

- Given a hypothesis H_0 and an alternative hypothesis H_a, we make a rule which is known as decision rule according to which we accept H_0 (i.e., reject H_a) or reject H_0 (i.e., accept H_a).
- For instance, if (H_0 is that a certain lot is good (there are very few defective items in it) against H_a) that the lot is not good (there are too many defective items in it), then we must decide the number of items to be tested and the criterion for accepting or rejecting the hypothesis.
- We might test 10 items in the lot and plan our decision saying that if there are none or only 1 defective item among the 10, we will accept H_0 otherwise we will reject H_0 (or accept H_a).
- This sort of basis is known as decision rule.

(d) Type I and Type II errors:

- In the context of testing of hypotheses, there are basically two types of errors we can make.
- We may reject H_0 when H_0 is true and we may accept H_0 when in fact H_0 is not true.
- The former is known as Type I error and the latter as Type II error.
- In other words, Type I error means rejection of hypothesis which should have been accepted and Type II error means accepting the hypothesis which should have been rejected.
- Type I error is denoted by **α** (alpha) known as **α** error, also called the level of significance of test; and Type II error is denoted by **β** (beta) known as **β** error.

In a tabular form the said two errors can be presented as follows:

Table 9.2

	Decision	
	Accept H_0	Reject H_0
H_0 (true)	Correct decision	Type I error (α error)
H_0 (false)	Type II error (β error)	Correct decision

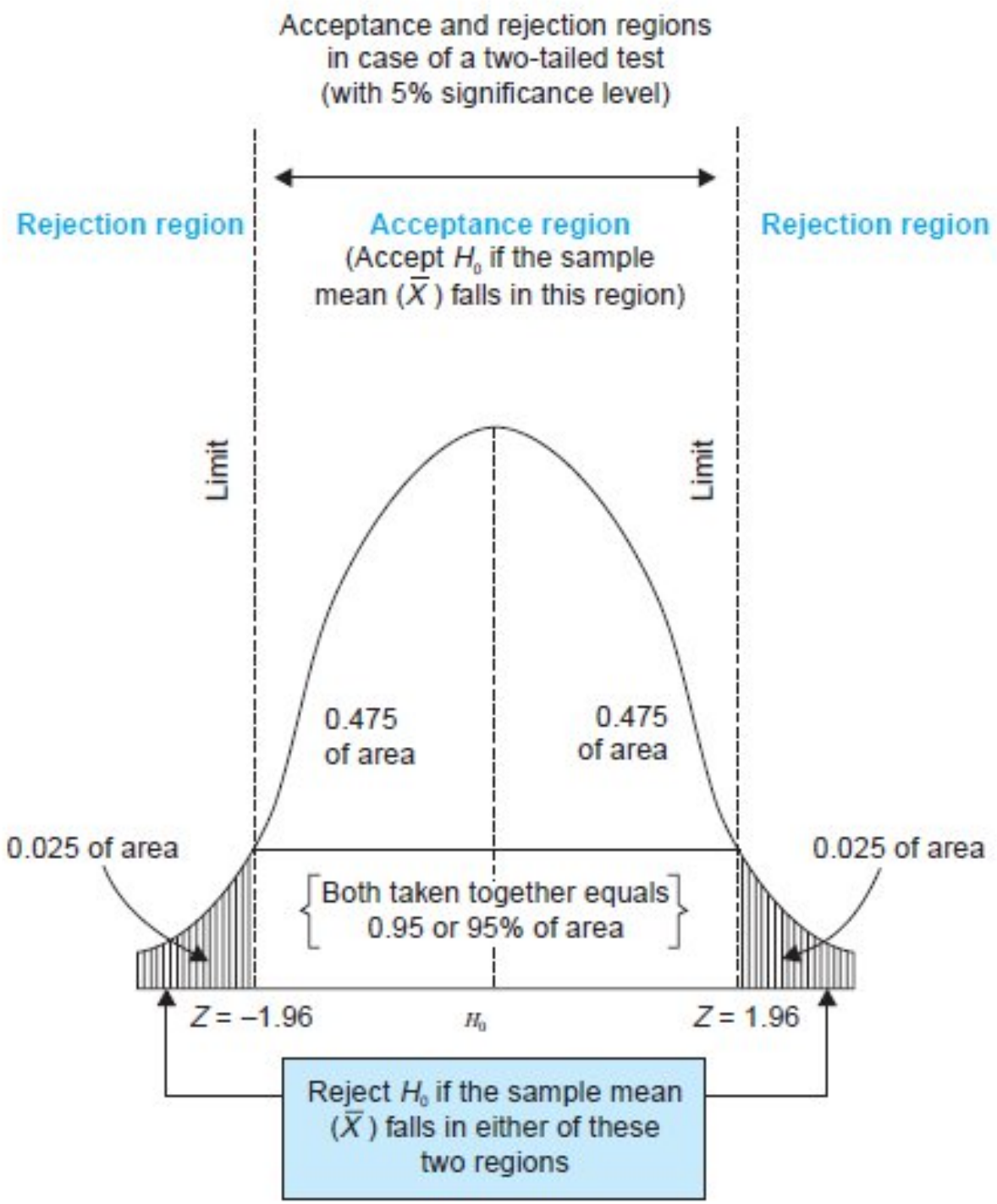
- The probability of Type I error is usually determined in advance and is understood as the level of significance of testing the hypothesis.
- If type I error is fixed at 5 per cent, it means that there are about 5 chances in 100 that we will reject H_0 when H_0 is true.
- We can control Type I error just by fixing it at a lower level.
- For instance, if we fix it at 1 per cent, we will say that the maximum probability of committing Type I error would only be 0.01.
- But with a fixed sample size, n, when we try to reduce Type I error, the probability of committing Type II error increases.
- Both types of errors cannot be reduced simultaneously. There is a trade-off between two types of errors which means that the probability of making one type of error can only be reduced if we are willing to increase the probability of making the other type of error.
- To deal with this trade-off in business situations, decision-makers decide the appropriate level of Type I error by examining the costs or penalties attached to both types of errors.

- If Type I error involves the time and trouble of reworking a batch of chemicals that should have been accepted, whereas Type II error means taking a chance that an entire group of users of this chemical compound will be poisoned, then in such a situation one should prefer a Type I error to a Type II error.
- As a result one must set very high level for Type I error in one's testing technique of a given hypothesis.
- Hence, in the testing of hypothesis, one must make all possible effort to strike an adequate balance between Type I and Type II errors.

(e) **Two-tailed and One-tailed tests:**

- In the context of hypothesis testing, these two terms are quite important and must be clearly understood.
- A two-tailed test rejects the null hypothesis if, say, the sample mean is significantly higher or lower than the hypothesized value of the mean of the population.
- Such a test is appropriate when the null hypothesis is some specified value and the alternative hypothesis is a value not equal to the specified value of the null hypothesis.
- Symbolically, the two tailed test is appropriate when we have $H_0 : m = m_{H0}$ and $H_a : m \neq \mu_{H0}$ which may mean $m > m_{H0}$ or $m < m_{H0}$.

- Thus, in a two-tailed test, there are two rejection regions*, one on each tail of the curve which can be illustrated as under:



Mathematically we can state:

Acceptance Region A : $|Z| \leq 1.96$

Rejection Region R : $|Z| \geq 1.96$

- If the significance level is 5 per cent and the two-tailed test is to be applied, the probability of the rejection area will be 0.05 (equally splitted on both tails of the curve as 0.025) and that of the acceptance region will be 0.95 as shown in the above curve.
- If we take $m = 100$ and if our sample mean deviates significantly from 100 in either direction, then we shall reject the null hypothesis; but if the sample mean does not deviate significantly from m , in that case we shall accept the null hypothesis.
- But there are situations when only one-tailed test is considered appropriate.
- A *one-tailed test* would be used when we are to test, say, whether the population mean is either lower than or higher than some hypothesised value.
- For instance, if our $H_0 : m = m_{H0}$ and $H_a : m \neq \mu_{H0}$, then we are interested in what is known as left-tailed test (wherein there is one rejection region only on the left tail) which can be illustrated as below:

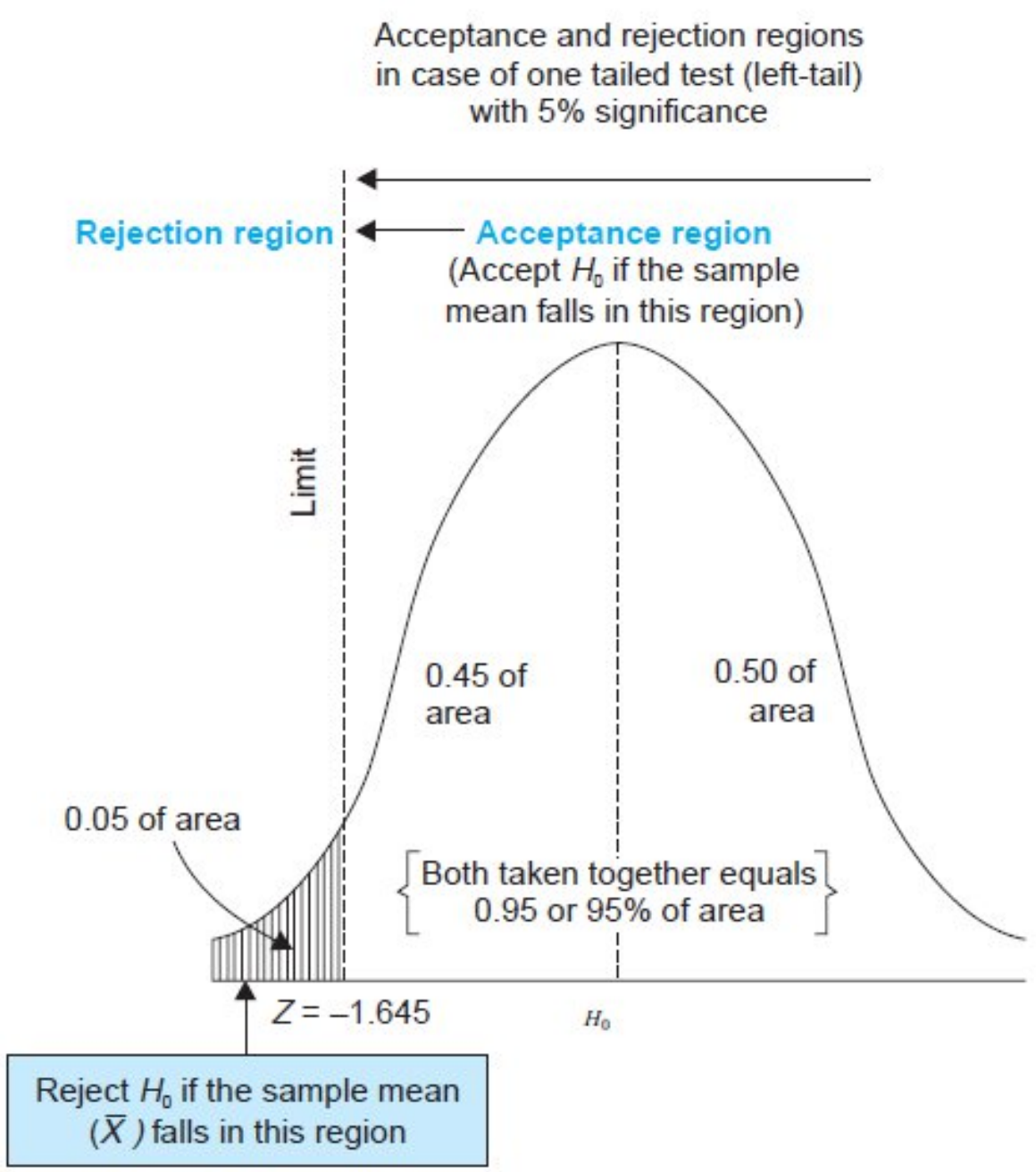

Acceptance and rejection regions
in case of one tailed test (left-tail)
with 5% significance
Rejection region
Acceptance region
(Accept H_0 if the sample
mean falls in this region)
Limit
0.45 of
area
0.50 of
area
0.05 of area
Both taken together equals
0.95 or 95% of area
$Z = -1.645$
H_0
Reject H_0 if the sample mean
($\bar{X}$) falls in this region

Fig. 9.2

Mathematically we can state:

Acceptance Region A : Z > -1.645

Rejection Region R : Z ≤ -1.645

- If our m = 100 and if our sample mean deviates significantly from100 in the lower direction, we shall reject H_0, otherwise we shall accept H_0 at a certain level of significance.
- If the significance level in the given case is kept at 5%, then the rejection region will be equal to 0.05 of area in the left tail as has been shown in the above curve.
- In case our $H_0 : m = m_{H0}$ and $H_a : m \neq \mu_{H0}$, we are then interested in what is known as one tailed test (right tail) and the rejection region will be on the right tail of the curve as shown below:

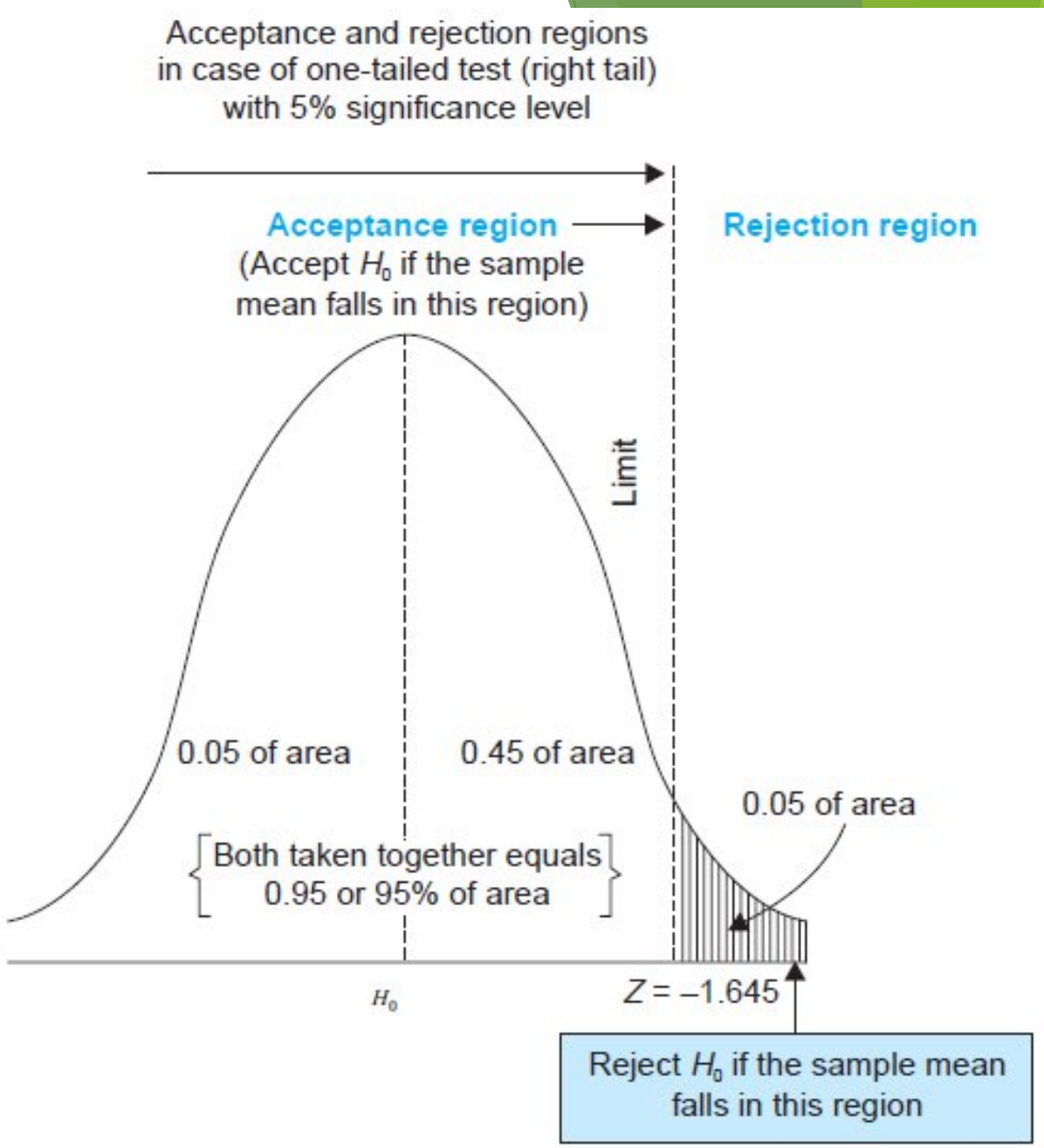

Acceptance and rejection regions in case of one-tailed test (right tail) with 5% significance level
Acceptance region
(Accept H_0 if the sample mean falls in this region)
Rejection region
Limit
0.05 of area
0.45 of area
0.05 of area
Both taken together equals 0.95 or 95% of area
H_0
$Z = -1.645$
Reject H_0 if the sample mean falls in this region

Fig. 9.3

Mathematically we can state:

$$Acceptance\ Region\ A : Z \leq -1.645$$

$$Rejection\ Region\ R : Z > -1.645$$

- If our $m = 100$ and if our sample mean deviates significantly from 100 in the upward direction, we
- shall reject H_0, otherwise we shall accept the same.
- If in the given case the significance level is kept at 5%, then the rejection region will be equal to 0.05 of area in the right-tail as has been shown in the above curve.
- It should always be remembered that accepting H_0 on the basis of sample information does not constitute the proof that H_0 is true.
- We only mean that there is no statistical evidence to reject it, but we are certainly not saying that H_0 is true (although we behave as if H_0 is true).

PROCEDURE FOR HYPOTHESIS TESTING

- To test a hypothesis means to tell (on the basis of the data the researcher has collected) whether or not the hypothesis seems to be valid. In hypothesis testing the main question is: whether to accept the null hypothesis or not to accept the null hypothesis?
- Procedure for hypothesis testing refers to all those steps that we undertake for making a choice between the two actions i.e., rejection and acceptance of a null hypothesis.
- The various steps involved in hypothesis testing are stated below:

(i) Making a formal statement:

- The step consists in making a formal statement of the null hypothesis (H_0) and also of the alternative hypothesis (*Ha*).
- This means that hypotheses should be clearly stated, considering the nature of the research problem.
- For instance, Mr. Mohan of the Civil Engineering Department wants to test the load bearing capacity of an old bridge which must be more than 10 tons, in that case he can state his hypotheses as under:

Null hypothesis H_0 : m = 10 tons

Alternative Hypothesis H_a: m > 10 tons

PROCEDURE FOR HYPOTHESIS TESTING

- Take another example. The average score in an aptitude test administered at the national level is 80.
- To evaluate a state's education system, the average score of 100 of the state's students selected on random basis was 75.
- The state wants to know if there is a significant difference between the local scores and the national scores.
- In such a situation the hypotheses may be stated as under:

  Null hypothesis H_0: $m = 80$

  Alternative Hypothesis H_a: $m \neq 80$
- The formulation of hypotheses is an important step which must be accomplished with due care in accordance with the object and nature of the problem under consideration.
- It also indicates whether we should use a one-tailed test or a two-tailed test.
- If *Ha* is of the type greater than (or of the type lesser than), we use a one-tailed test, but when *Ha* is of the type "whether greater or smaller" then we use a two-tailed test.

PROCEDURE FOR HYPOTHESIS TESTING

(ii) *Selecting a significance level:*

- The hypotheses are tested on a pre-determined level of significance and as such the same should be specified.
- Generally, in practice, either 5% level or 1% level is adopted for the purpose.
- The factors that affect the level of significance are: (a) the magnitude of the difference between sample means; (b) the size of the samples; (c) the variability of measurements within samples; and (d) whether the hypothesis is directional or non-directional (A directional hypothesis is one which predicts the direction of the difference between, say, means).
- In brief, the level of significance must be adequate in the context of the purpose and nature of enquiry.

(iii) *Deciding the distribution to use:*

- After deciding the level of significance, the next step in hypothesis testing is to determine the appropriate sampling distribution.
- The choice generally remains between normal distribution and the *t*-distribution.
- The rules for selecting the correct distribution are similar to those which we have stated earlier in the context of estimation.

PROCEDURE FOR HYPOTHESIS TESTING

(iv) *Selecting a random sample and computing an appropriate value:*

- ❖ Another step is to select a random sample(s) and compute an appropriate value from the sample data concerning the test statistic utilizing the relevant distribution.
- ❖ In other words, draw a sample to furnish empirical data.

(v) *Calculation of the probability:*

- One has then to calculate the probability that the sample result would diverge as widely as it has from expectations, if the null hypothesis were in fact true.

(vi) *Comparing the probability:*

- Yet another step consists in comparing the probability thus calculated with the specified value for α , the significance level.
- If the calculated probability is equal to or smaller than the α value in case of one-tailed test (and $\alpha/2$ in case of two-tailed test), then reject the null hypothesis (i.e., accept the alternative hypothesis), but if the calculated probability is greater, then accept the null hypothesis.
- In case we reject H_0, we run a risk of (at most the level of significance) committing an error of Type I, but if we accept H_0, then we run some risk (the size of which cannot be specified as long as the H_0 happens to be vague rather than specific) of committing an error of Type II.

FLOW DIAGRAM FOR HYPOTHESIS TESTING

The above stated general procedure for hypothesis testing can also be depicted in the from of a flowchart for better understanding as shown in Fig. 9.4

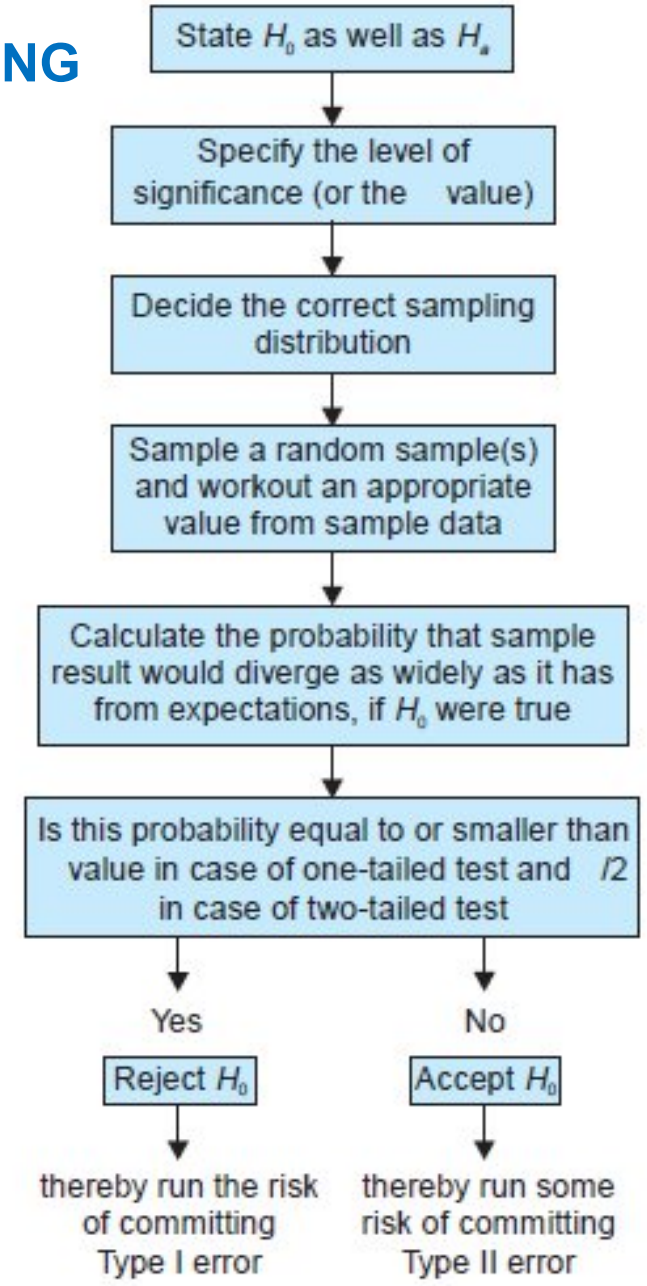

State H_0 as well as H_a
Specify the level of significance (or the value)
Decide the correct sampling distribution
Sample a random sample(s) and workout an appropriate value from sample data
Calculate the probability that sample result would diverge as widely as it has from expectations, if H_0 were true
Is this probability equal to or smaller than value in case of one-tailed test and /2 in case of two-tailed test
Yes
No
Reject H_0
Accept H_0
thereby run the risk of committing Type I error
thereby run some risk of committing Type II error

Fig. 9.4

MEASURING THE POWER OF A HYPOTHESIS TEST

- As stated above we may commit Type I and Type II errors while testing a hypothesis.
- The probability of Type I error is denoted as a (the significance level of the test) and the probability of Type II error is referred to as b.
- Usually the significance level of a test is assigned in advance and once we decide it, there is nothing else we can do about a.
- But what can we say about b? We all know that hypothesis test cannot be foolproof; sometimes the test does not reject *H*0 when it happens to be a false one and this way a Type II error is made.
- But we would certainly like that b (the probability of accepting *H*0 when *H*0 is not true) to be as small as possible.

MEASURING THE POWER OF A HYPOTHESIS TEST

- Alternatively, we would like that 1 – b (the probability of rejecting *H*0 when *H*0 is not true) to be as large as possible. If 1 – b is very much nearer to unity (i.e., nearer to 1.0), we can infer that the test is working quite well, meaning thereby that the test is rejecting *H*0 when it is not true and if 1 – b is very much nearer to 0.0, then we infer that the test is poorly working, meaning thereby that it is not rejecting *H*0 when *H*0 is not true.

- Accordingly 1 – b value is the measure of how well the test is working or what is technically described as the *power of the test*. In case we plot the values of 1 – b for each possible value of the population parameter (say m , the true population mean) for which the *H*0 is not true (alternatively the *H*a is true), the resulting curve is known as the power curve associated with the given test.

MEASURING THE POWER OF A HYPOTHESIS TEST

- Thus power curve of a hypothesis test is the curve that shows the conditional probability of rejecting *H*0 as a function of the population parameter and size of the sample.
- The function defining this curve is known as the *power function*. In other words, the power function of a test is that function defined for all values of the parameter(s) which yields the probability that *H*0 is rejected and the value of the power function at a specific parameter point is called the power of the test at that point.
- As the population parameter gets closer and closer to hypothesized value of the population parameter, the power of the test (i.e., 1 – b) must get closer and closer to the probability of rejecting *H*0 when the population parameter is exactly equal to hypothesised value of the parameter.
- We know that this probability is simply the significance level of the test, and as such the power curve of a test terminates at a point that lies at a height of a (the significance level) directly over the population parameter.

MEASURING THE POWER OF A HYPOTHESIS TEST

- Closely related to the power function, there is another function which is known as the *operating characteristic function* which shows the conditional probability of accepting $H0$ for all values of population parameter(s) for a given sample size, whether or not the decision happens to be a correct one.
- If power function is represented as H and operating characteristic function as L, then we have $L = 1 - H$. However, one needs only one of these two functions for any decision rule in the context of testing hypotheses.
- How to compute the power of a test (i.e., 1 – b) can be explained through examples.